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INCONSISTENCIES IN MOTION

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I. INTRODUCTION: THE PROBLEM OF CHANGE

A moment ago you were not reading this paper; now you are. Things have changed. But what exactly is change? This is a thorny old question. It is one, however, I wish to address. I will proceed as follows. I will give two answers to the question of what change is. One is the orthodox, consistent, Russellean view. The other is the more embryonic, inconsistent, Hegelean view. I will argue that the orthodox view is not at all without its problems, and that the Hegelean view can be developed in a rigorous and coherent manner which, I will suggest, presents a plausible rival to the orthodox view.

In what I have to say, I will restrict myself to the issue of change of place with respect to time: motion (and I shall always speak of a body in motion, rather than the more accurate pair of bodies in relative motion). I do this not because there is something peculiar about motion; quite the reverse: it is the paradigm of change. Fixing on it will allow the discussion a precision it would otherwise lack; and what I say concerning motion can easily be generalised to other kinds of change.

II. THE ORTHODOX ANSWER

First the orthodox answer. The answer is not orthodox in the sense that most philosophers have endorsed it. This history of philosophy in fact shows little consensus on the issue. It is orthodox in that it is now, I am sure, the received view. It was first formulated clearly and precisely by Bertrand Russell. According to Russell motion consists *merely* in the occupation of different places at different times.[1]

> Motion consists in the fact that, by the occupation of a place at a time, a correlation is established between places and times; when different times, throughout any period however short, are correlated with different places, there is motion; when different times, throughout some period however short, are all correlated with the same place, there is rest.

Thus, what makes something in motion at an instant is simply that at arbitrarily close instants it can be found at a different place. Russell is actually slightly inconsistent, since after giving this definition he permits that something may be momentarily at rest if its position derivative with respect to time is zero. This is quite compatible with its being in motion in the official sense. However, this inconsistency need not concern us here.

III. PROBLEMS WITH THE ORTHODOX ANSWER

Despite the fact that this view of the subject is the received one, it faces some not inconsiderable objections. None of them is perhaps a guaranteed knock-down argument—in philosophy there are very few such arguments. However they certainly show that the orthodox account does not have it all its own way. It is certainly not the universal panacea to the discomforts that people have felt about change, as those such as Russell had hoped it would be. Here are two objections.

α) First, it follows from Russell's definition that there is no such thing as an *intrinsic* state of change. If one had a body in motion and took, as it were, a logical "picture" of it at a certain instant, the "picture" one would obtain would be no different to one of a similar body in the same place but at rest.

Of course an object in motion can have an instantaneous non-zero velocity. It would be wrong, however, to think that this differentiates it intrinsically from a static body. For to say that it has an instantaneous velocity at t_0 is just to say that

$$df/dt \neq 0 \text{ at } t = t_0$$

(where f is the function which gives its location with respect to time), i.e., that:

$$\lim_{\epsilon \to 0} \frac{f(t_0+\epsilon) - f(t_0)}{\epsilon} \neq 0$$

The lim-operator here in effect quantifies over all instants around t_0. Hence an instantaneous velocity is essentially relational.

Russell in fact points out that there is no intrinsic state of motion, and even revels in it:[2]

> [Zeno's arrow argument] denies that there is such a thing as a *state* of motion...This has usually been thought so monstrous a paradox as scarcely to deserve serious attention. To my mind I must confess, it seems a very plain statement of a very elementary fact, and its neglect has, I think, caused the quagmire in which the philosophy of change has long been immersed...Change does not involve a state of change.

On this account, then, there is no such thing as an intrinsic state of motion. The instantaneous states of a body in motion are qualitatively indistinguishable from the corresponding states of a body at rest. In picturesque terms, motion is rather like a sequence of photographic stills (albeit a dense and continuous one) shown so fast that the body appears to move. But this conception of motion jars against our intuitive notion of motion as a genuine flux. A journey is not a sequence of states indistinguishable from rest states, even a lot of them close together. If God were to take temporal slices of an object at rest in different places, and string them together in a continuous fashion, he would not have made the object move.

One way of bringing this point home[3] is this. Suppose that the universe were a Laplacean universe whose state at any time is determined by the state at any (prior) time. Then the orthodox account of change is impossible. For the instantaneous state of an object in the universe cannot even determine whether it is at rest or in motion, and hence whether it is at the same or a different point at a subsequent time. (Recall that the velocity—or momentum—of an object is not defined by its instantaneous state.) Now I am certainly not insisting that the universe is Laplacean. (It is not.) But it is a curious theory which rules this out *a priori*.

β) The non-intrinsic nature of motion can be exploited to produce a second argument against the orthodox account. This is done, as might be expected, with the help of Zeno. Zeno's paradoxes have long plagued accounts of change. Of the four usually cited, I think perhaps the most profound is that of the arrow. Certainly it is this which provides problems for the orthodox account of change.

Consider an object in uniform motion, say, the tip of an arrow, travelling from A to B, and take an instant, t_0, of its motion. At t_0 the arrow advances not on its journey towards B. (If it did make some headway on its journey, this would take time. The temporal stretch involved would not therefore be an instant.) Thus at $t = t_0$, total progress made equals zero. But a temporal interval $a \leq t \leq b$ is made up of such points. It would seem therefore that since no progress is made in any basic part of the interval $[a, b]$, no progress can be made in the whole, i.e. the arrow never makes any progress on its journey at all. This is absurd.

The received answer to this one of Zeno's paradoxes is closely connected with the orthodox account of change.[4] In fact, the orthodox account leaves very little room to manoeuvre. For up until the very last step the conclusions of the reasoning are in agreement with this account. At each instant of the motion the arrow *does* make no advance on its journey: it is qualitatively indistinguishable from a body at rest. The only possibility for avoiding the paradox is a denial of the final step. Even given that in each instant the arrow makes no progress on the journey, in the sum of all instants, it does. The whole is greater than the sum of its parts. Technically, though the measure (= length) of the points traversed in an instant is zero, the measure of points traversed in a sum of instants may be non-zero, (provided there are sufficiently, i.e. uncountably, many points). Of course, to deny this step is to say where the argument fails, but it is hardly to solve the paradox. For the denial of the principle involved in the final step of the argument seems just as puzzling as the conclusion of the paradox. How can going somewhere be composed of an aggregate of going nowheres?

One should perhaps separate a technical mathematical question here from a philosophical one. Technically, we can represent the length of a certain set of points by a measure function σ. If we define a measure function on the real line in one of a number of standard ways, we can show that if Σ is any finite (or even countable) set of points $\sigma(\Sigma)=0$, whilst if Σ is an interval $[a, b]$, $\sigma(\Sigma)=b-a$. Thus the length of the set of points traversed at an instant (which is a singleton set) is zero. But the length of a set of points traversed in an interval of time (which is itself an interval) has non-zero measure.

That one can prove a small mathematical theorem or two is one thing. However it does not ease the discomfort one finds (at least that I find) when one tries to understand what is going on physically, when one tries to understand how the arrow actually achieves its motion. At any point in its motion it advances not at all; yet in some apparently magical way, in a collection of these it advances. Now a sum of nothings (even infinitely many nothings) is nothing. So how does it do it?

IV. The Hegelean Answer

The above problems of the orthodox account make it desirable to consider alternatives. So let us look at one. This account of motion is that of Hegel. It is almost certainly a good deal older. It is arguable that it is to be found in Heraclitus, who certainly influenced Hegel.[5] Hegel himself seems to attribute it to Zeno.[6] This is historically problematic. However undoubtedly Zeno's paradoxes of motion played a crucial role in Hegel's thought. Anyway Hegel gives probably the clearest statement of the view, so let us stick to that. Unlike Russell, Hegel did hold that motion is intrinsic: there is an instantaneous difference between a moving body and a stationary body. What this is, is best left for Hegel to say for himself:[7]

> Something moves not because at one moment it is here and at another there, but because at one and the same moment it is here and not here...motion itself is contradiction's immediate existence.

Hegel is not denying that if something is in motion it will be at different places at different times. Rather, the point is that this is not *sufficient* for it to be in motion. It would not distinguish it from a body occupying different places at different times but *at rest* at each of these instants. What is sufficient for it to be in motion at a certain time is for it both to occupy and not occupy a certain place.

Hegel's account of motion would not seem to have a great deal going for it. For a start, it is inconsistent—which is enough to put most people off. It is also obscure, and it is not really clear how it is meant to relate to the more familiar aspects of motion such as change of place. In particular it does not seem to relate in any way to the canonical representation of motion, by functional equations, in science and applied mathematics. I (like Russell) am enough of a holist to think that our philosophical understanding of change and our scientific understanding must be compatible. For change is, and always has been, one problem with both philosophical and scientific aspects. One cannot divide them. It is the great strength of the orthodox account that it coheres with the canonical representation of change. Thus, for example, an equation representing a motion, $x = f(t)$, just seems to encode the idea of different places at different times: it merely records the functional correlation. By contrast, Hegel's view seems to have no bearing on this.

For these reasons few would now take Hegel's view of motion seriously. Despite this I think it can be developed into a coherent rival to the Russellean view and one which is in many ways more attractive. First of all let us consider why exactly Hegel thought that motion realized contradictions. The idea is something like this: consider a body in motion, say a point particle. At a certain instant, t, it occupies a point in space, x, and, since it is there, it is nowhere else. But now consider a time very, very close to t, t', removed, say, by a magnitude in the order of Planck's constant. Let us suppose that over such small intervals of time it is impossible to localise the body. Thus the body is equally at the place it occupies at t', x' ($\neq x$). Hence at this instant the body is both at x' and not at x'. This is essentially why Hegel thought that motion realises a contradiction.

Of course there is more to the story than this. For Hegel gives a reason why the moving body cannot be localised. The reason derives from Hegel's view of the continuum. Essentially it is that in a continuum distinct points themselves merge together. Thus the reason that we can't localise the body just to t is that t may not itself be "localisable." Thus, as Hegel puts it:

> [When a body is moving] there are three different places: the present place, the place about to be occupied, and the place which has just been vacated; the vanishing of the dimension of time is paralysed. But at the same time there is only *one* place, a universal of these places, which remains unchanged through all the changes; it is duration existing immediately in accordance with its Notion, and as such it is Motion.[8]

Hegel's view of the continuum is a fascinating one involving a number of issues. One is the notion of a variable (point) as it was conceived of in eighteenth century calculus. Another is Hegel's view that such a point is the contradictory unity of the discrete and the continuous.[9] However, this is not the place to go into that. Neither do we have to endorse the Hegelean conception of the continuum. Let us just accept, tentatively, say as a speculative hypothesis, that the localisation of an object is impossible over very small temporal distances, or, to be a bit more precise, the following principle, which we will call the *spread hypothesis*: A body cannot be localised to a point it is occupying at an instant of time, but only to those points it occupies in a small neighbourhood of that time. This is not yet completely precise, but I will give a rigorous formulation once I have introduced enough semantical machinery to do so.

The spread hypothesis may be strange. Yet we are now accustomed to the idea that very strange things happen at very small orders of spatio-temporal magnitude. And the spread hypothesis is no stranger than many such things.

V. State Descriptions of Motion

So much for an informal discussion of the Hegelean account of motion. It is now time to put the theory on a rigorous basis by showing how it can be represented in formal logical semantics, and this means that we must tackle the question of consistency. Motion, according to Hegel, is inconsistent. Thus we will not get very far whilst we work with a logical theory, such as that of classical or intuitionist logic, which makes a nonsense of inconsistency. We must have a way of handling inconsistent situations and true contradictions. In the last twenty years several such logical theories have been developed.[10] These are paraconsistent. They are not all equal in merit. However, any one of them could be used for the present application. Let ν be a semantical evaluation. We will write "ν assigns the formula A the value true" as "$\nu(A) = T$". The crucial thing about paraconsistent evaluations is that it may well be the case that for some formula A, $\nu(A) = T$ and $\nu(\neg A) = T$.

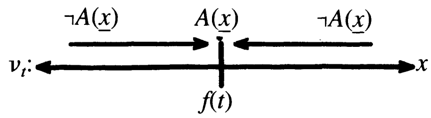
To handle motion, we need to consider semantic evaluations of the following simple propositional language. The atomic formulas of the language, P, are of the form "object x is at point r." For simplicity, we will assume that the places in question are on a one-dimensional continuum, and hence can be specified by a real number, r. Again for simplicity, we will assume that each real, r, has a name in the language, $\underline{r}$. (We could avoid this by talking in terms of satisfacton rather than truth.) The set of formulas, F, is the closure of P under negation $\neg$, conjunction $\wedge$, and disjunction $\vee$.

This machinery is not quite sufficient to handle the problem of motion: for the formulas of F change their truth values over time. Hence we need to consider the semantics of a tense logic. Semantically, a tense logic comprises a set X, thought of as instants of time, with a relation R, thought of as the relation of temporal order. The other component is a function ν: $X \rightarrow \cdot$, which we will call a *state description*, such that for all $t \epsilon X$, ν_t is a semantical evaluation. Thus ν_t specifies which sentences are true, at time t.[11] Of course the important case of a tense logic is when $X = \mathrm{IR}$, the real line, and R is the usual ordering on IR, $<$. I am well aware that one may raise philosophical doubts about the adequacy of the real line to represent the temporal continuum, but scientifically it is not at issue. Henceforth I shall restrict myself to semantic structures where time is IReal.

We are now in a position to make the discussion of motion of the previous section precise. Consider an object, a, which moves according to the equation $x = f(t)$. Let us first describe the Russellean state description of such a body. Let us write the sentence "a is at point $\underline{r}$" as: $A(\underline{r})$. Then the Russellean state description of a is the evaluation ν such that

$$(\alpha) \qquad \begin{aligned} \nu_t(A(\underline{r})) &= T \text{ if } r = f(t) \\ \nu_t(\neg A(\underline{r})) &= T \text{ if } r \neq f(t). \end{aligned}$$

Given that ν is consistent, this completely specifies it. For any t we could illustrate ν_t as follows:



However, once we move to paraconsistent evaluations (α) is no longer a complete specification. For example it does not specify whether $\nu_t(A(\underline{r})) = T$, or not if $r \neq f(t)$. We will now exploit this.

First, let us give a precise formulation of the spread hypothesis of the last section.

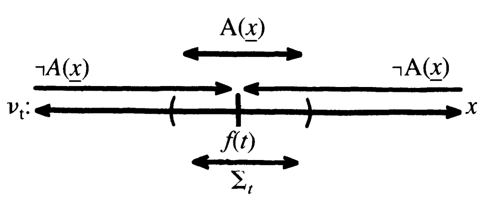
Spread Principle:

> For any motion, $x = f(t)$, and any instant of time, t, there is an interval around t, θ_t such that for all $t' \in \theta_t$, $\nu_t(A(f(\underline{t'})) = T$.[12]

If we now apply the spread principle, it tells us that for all $t' \in \theta_t$

(β) $\qquad \nu_t(A(\underline{r})) = T$ if $r = f(t')$

If we add this information to (α) we can depict the situation thus:



Where $\Sigma_t = \{x | \exists t' \in \theta_t x = f(t')\}$.
This is still not a complete specification of ν, since we have as yet said nothing about, e.g. $\nu_t\ (A(\underline{x}))$ if $x \notin \Sigma_t$. However, let us round off the picture by specifying that nothing takes the value T unless it is forced to by either the equation of motion or the spread principle. We will call this state description the Hegelean state description of the motion, and the above picture is an excellent depiction of it. A glance at it show that provided Σ_t is not degenerate then at the instant t a number of contradictions are realised. For all $r \in \Sigma_t - \{f(t)\}$, $\nu_t(A(\underline{r}) = T$ and $\nu_t(\neg A(\underline{r})) = T$. Thus is the contradictory nature of motion made manifest.

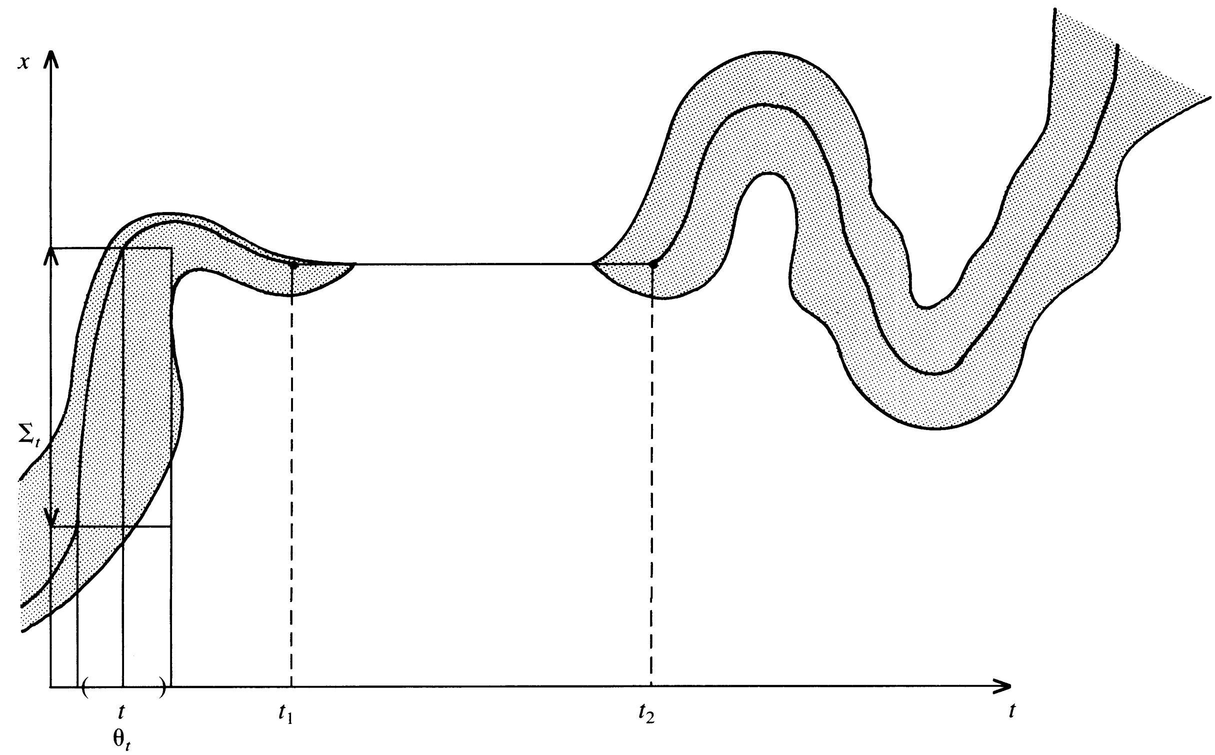
Σ_t may be degenerate for one of two reasons. The first is that θ_t is itself degenerate, i.e. $\theta_t = \{t\}$. The other is that although θ_t is not itself degenerate, f is constant over it. I have so far said nothing about θ_t. Clearly there is more to be said. However for the general purposes at hand we can assume that θ_t is rarely degenerate (perhaps only when $df/dt = 0$) and is generally small. In particular, if a body occupies the same spot for any reasonable length of time (longer than θ_t), then Σ_t will be degenerate, and no contradictions will be forthcoming. Hence, rest corresponds to a non-contradictory state. (There may be a few singular cases that are worth analysing but I will not go into this now.) To round off the discussion, consider a body whose equation of motion is $x = f(t)$, and suppose that $f(t)$ is constant between t_1 and t_2. Then we might attempt to draw the whole contradictory state of motion thus, where the shaded area represents "contradictory space-time points."[13] (See figure on page 344.)

To summarise: according to this view, to be in motion is to occupy more than one place (in fact a continuum of places) at the same time, and hence to be and not to be in some places.

VI. The Merits of Hegel's Answer

This construction shows us that sense can indeed be made of Hegel's account of change. In particular the defects of the account I noted at the beginning of Section IV are overcome. The account is not at all obscure: it is rigorous and precise. It is still inconsistent, but it is clear how this inconsistency is to be handled in a satisfactory way. (I do not deny that there are general philosophical problems concerning the notion of true contradictions, but these are not specific to Hegel's account of motion.) And most importantly, it is quite compatible with the standard mathematical representation of change. An equation of motion, such as $x = f(t)$ still captures the idea that at any t the object is at $f(t)$. It is just that there is more to change than this. It might be elsewhere too!

It will now pay us to see how Hegel's account fares with respect to the objections I brought against Russell's account in Section III. The first objection was that according to Russell, there is no such thing as an intrinsic state of change. (The cinematographic objection.) As we have seen, this is no problem for Hegel's account. For there is an



intrinsic difference. The instantaneous moving state is a contradictory state, whilst the instantaneous rest state is not. The second objection to the Russellean account that I considered was Zeno's paradox of the arrow. The Hegelean account may be taken to locate a fault in the version of Zeno's argument that I gave, but at a point different to the one Russell locates. For according to the argument, at a particular point of time, t, the object occupies only a single place, whence it follows that it advances not on its journey during that instant, i.e. that the set of points occupied during that time is of measure zero. However, as we have seen, given the spread hypothesis, it is not true that a moving body occupies merely a single point. In particular, at an instant, t, it occupies all those points in Σ_t. And as we have seen this is, in general, not a singleton. Moreover, provided that the equation of motion is continuous, this is an interval and hence has non-zero measure. Thus, advance *is* made during a single instant and hence during the aggregate of instants. In picturesque terms we might say that at any instant of motion, the object still is where it has already left, and already is where it has not yet arrived!

At any rate, the arrow paradox is solved, and hence both the problems of the orthodox account are handled by the Hegelean one. Whether these are decisive advantages, it would be premature to judge. Hegel's account is of a kind very unfamiliar to twentieth century philosophers and logicians, and its implications and ramifications will take time to mull over.

VII. Conclusion

At any rate, I have shown that Hegel's account of motion can be put on a rigorous basis and also, I think, that it compares very favourably with the orthodox account. The Hegelean account also invites a good deal of further research. For, as we have seen, the contradictions that arise in motion have a determinate logical structure. What exactly this is, I have done no more than indicate. Many further questions pose themselves. For example, is the length of the interval involved in the spread principle, θ_t, constant or does it depend, e.g., on the velocity of the motion, and if so, how? What is its physical significance?[14] It would be foolish to try to treat these questions here. However I cannot resist a (perhaps rather fanciful) speculation. According to quantum theory, given any particle, there is an uncertainty in its location at any time t. This is not surprising if indeed the particle is not located at a single point but is "spread out" over the whole interval Σ_t. Perhaps the measure of Σ_t, $\sigma(\Sigma_t)$, just is the uncertainty in the location of the object at t. Perhaps quantum indeterminacies are fundamentally caused by the inconsistencies in motion and, in particular, by the spread given by the spread principle. This suggestion at least allows us to give physical significance to the interval involved in the spread principle. For in general the momentum, p, of an object is in a continuous state of change too. Hence by exactly the same reasoning we can conclude that the momentum at t is spread out over a range Π_t. Heisenberg's uncertainty principle then gives us that:

$$\sigma(\Sigma_t).\ \sigma(\Pi_t) \geq \hbar$$[15].

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NOTES

1. Russell [1903], Section 447.
2. Russell [1903], pp. 351, 350, xxxiii. I have rearranged the quotations without, I think, doing Russell an injustice. The italics are original.
3. For which I am grateful to Michael Tooley.
4. It is given by Russell [1903], Section 332.
5. See Hegel [1840], Vol. 1, Ch. 1, Section D.
6. See Hegel [1840], Vol. 1, Ch. 1, Section C.4.
7. Hegel [1812], p. 440. (Sentence order changed.)
8. Hegel [1830], p. 43.
9. Hegel [1840], p. 268. "To us there is no contradiction in the idea that the here of space and the now of time [i.e. variable points in a continuum] are considered as a continuity or length; but their notion is self-contradictory. Self-identity or continuity is absolute cohesion, the destruction of all difference, of all negation, of all being for self; the point on the contrary, is pure being-for-self, absolute self-distinction and the destruction of all identity and all connection with what is different."
10. For a survey of these see Priest and Routley [1984]. A more detailed investigation can be found in Priest and Routley [1983].
11. We can, if we wish, extend the propositional language in the usual way with the tense operators, P and F, and supply them with semantic conditions. This is unnecessary here. Details can be found in Priest [1982].
12. θ_t may of course depend on f.
13. By which I mean no more than that they are the points $\langle x, t\rangle$ for which $\nu_t(A(\underline{x})) = \nu_t(\neg A(\underline{x})) = T$. It should not be forgotten that this is relative to a frame of reference. An absolute statement could be obtained by taking the A sentences to express a relation between the object, a, and an object at the origin of the frame of reference, b. The Hegelean view of motion is not committed to an absolute view of space.
14. A further question concerns the symmetry of θ_t about t. It might be pointed out (as David Lewis did to me) that though, on the account given, the *state* of motion is intrinsic to the instant, the direction of motion is not. This can be remedied, if it need be, by building an asymmetry into θ_t, for example, by making the leading edge open and the trailing edge closed.
15. This paper was read at a meeting of the Australasian Association of Philosophy in Adelaide 1983.

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